1. **Gaussian distribution**
   (a) Use `randn` to generate 1000000 normally (zero mean, unit variance) distributed random numbers.
   (b) Plot a properly normalized histogram of these random numbers.
   (c) Compare the histogram with the probability density of the Gaussian distribution

   $$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

   where μ is the mean and σ^2 is the variance of the Gaussian distribution.
   (d) Generate Gaussian distributed random numbers with mean $\mu = 2$ and standard deviation $\sigma = \frac{1}{2}$.
2. **Covariance and correlation coefficient**
   (a) Generate two vectors x and z with Gausian distributed random numbers.
   (b) Compute y as a linear combination of x and z according to

   $$y = r \cdot x + \sqrt{1 - r^2} \cdot z$$

   where r is a parameter $-1 \leq r \leq 1$. What does r do?
   (c) Plot a scatter plot of y versus x for about 10 different values of r. What do you observe?
   (d) Also compute the covariance matrix and the correlation coefficient matrix between x and y (functions `cov` and `corrcoef`). How do these matrices look like for different values of r? How do the values of the matrices change if you generate x and z with larger variances?
   (e) Do the same analysis (Scatter plot, covariance, and correlation coefficient) for

   $$y = x^2 + 0.5 \cdot z$$

   Are x and y really independent?
3. **Principal component analysis**
   (a) Generate pairs (x, y) of Gaussian distributed random numbers such that all x values have zero mean, half of the y values have mean $+d$ and the other half mean $-d$, with $d \geq 0$.
   (b) Plot scatter plots of the pairs (x, y) for $d = 0$, 1, 2, 3, 4 and 5. Also plot a histogram of the x values.
   (c) Apply PCA on the data and plot a histogram of the data projected onto the PCA axis with the largest eigenvalue. What do you observe?