1. **Homogeneous Poisson process** A homogeneous Poisson process of rate λ (measured in Hertz) is a point process where the probability of an event is independent of time t and independent of previous events. The probability P of an event within a bin of width Δt is

$$P = \lambda \cdot \Delta t$$

for sufficiently small Δt.

(a) Write a function that generates n homogeneous Poisson spike trains of a given duration T_{max} with rate λ.

(b) Using this function, generate a few trials and display them in a raster plot.

(c) Write a function that extracts a single vector of interspike intervals from the spike times returned by the first function.

(d) Write a function that plots the interspike-interval histogram from a vector of interspike intervals. The function should also compute the mean, the standard deviation, and the CV of the intervals and display the values in the plot.

(e) Compute histograms for Poisson spike trains with rate $\lambda = 100$ Hz. Play around with T and n and the bin width (start with 1 ms) of the histogram. How many interspike intervals do you approximately need to get a "nice" histogram? How long do you need to record from the neuron?

(f) Compare the histogram with the true distribution of intervals T of the Poisson process

$$p(T) = \lambda e^{-\lambda T}$$

for various rates λ.

(g) What happens if you make the bin width of the histogram smaller than Δt used for generating the Poisson spikes?

(h) Plot the mean interspike interval, the corresponding standard deviation, and the CV as a function of the rate λ of the Poisson process. Compare the simulations with the theoretical expectations for the dependence on λ.

(i) Write a function that computes serial correlations for the interspike intervals for a range of lags. The serial correlations ρ_k at lag k are defined as

$$\rho_k = \frac{\langle (T_{i+k} - \langle T \rangle)(T_i - \langle T \rangle) \rangle}{\langle (T_i - \langle T \rangle)^2 \rangle} = \frac{\mathrm{cov}(T_{i+k}, T_i)}{\mathrm{var}(T_i)}$$

Use this function to show that interspike intervals of Poisson spikes are independent.

(j) Write a function that generates from spike times a histogram of spike counts in a count window of given duration W. The function should also plot the Poisson distribution

$$P(k) = \frac{(\lambda W)^k e^{\lambda W}}{k!}$$

for the rate λ determined from the spike trains.

(k) Write a function that computes mean count, variance of count and the corresponding Fano factor for a range of count window durations. The function should generate tow plots: one plotting the count variance against the mean, the other one the Fano factor as a function of the window duration.