

Emergent intensity invariance in a physiologically inspired model of the grasshopper auditory system

Jona Hartling, Jan Benda

1 Exploring a grasshopper's sensory world

Strong dependence on acoustic signals for ranged communication

- Diverse species-specific sound repertoires and production mechanisms
- Different contexts/ranges: Stridulatory, mandibular, wings, walking sounds
- Mate attraction/evaluation, rival deterrence, loss-of-signal predator alarm
- Elaborate acoustic behaviors co-depend on reliable auditory perception

Songs = Amplitude-modulated (AM) broad-band acoustic signals

- Generated by stridulatory movement of hindlegs against forewings
- Shorter time scales: Characteristic temporal waveform pattern
- Longer time scales: High degree of periodicity (pattern repetition)
- Sound propagation: Signal intensity varies strongly with distance to sender
- Ectothermy: Temporal structure warps with temperature
- Sensory constraints imposed by properties of the acoustic signal itself

Multi-species, multi-individual communally inhabited environments

- Temporal overlap: Simultaneous singing across individuals/species common
- Frequency overlap: No/hardly any niche speciation into frequency bands
- "Biotic noise": Hetero-/conspecifics ("Another one's songs are my noise")
- "Abiotic noise": Wind, water, vegetation, anthropogenic
- Effects of habitat structure on sound propagation (landscape - soundscape)
- Sensory constraints imposed by the (acoustic) environment

Cluster of auditory challenges (interlocking constraints → tight coupling):

From continuous acoustic input, generate neuronal representations that...

- 1)...allow for the separation of relevant (song) events from ambient noise floor
- 2)...compensate for behaviorally non-informative song variability (invariances)
- 3)...carry sufficient information to characterize different song patterns, recognize the ones produced by conspecifics, and make appropriate behavioral decisions based on context (sender identity, song type, mate/rival quality)

How can the auditory system of grasshoppers meet these challenges?

- What are the minimum functional processing steps required?
- Which known neuronal mechanisms can implement these steps?
- Which and how many stages along the auditory pathway contribute?

→ What are the limitations of the system as a whole?

How can a human observer conceive a grasshopper's auditory percepts?

- How to investigate the workings of the auditory pathway as a whole?
- How to systematically test effects and interactions of processing parameters?
- How to integrate the available knowledge on anatomy, physiology, ethology?
- Abstract, simplify, formalize → Functional model framework

Precursor work for model construction (special thanks to authors):

Linear-nonlinear modelling of behavioral responses to artificial songs

- Feature expansion as implemented in our model: Major contribution!
- Bank of linear filters, nonlinearity, temporal integration, feature weighting
- Clemens and Hennig 2013 (crickets)
- Clemens and Ronacher 2013 (grasshoppers)
- Ronacher et al. 2015

Own advancements/key differences:

- 1) Used boxcar functions as artificial "songs" (focus on few key parameters)
→ Now actual, variable songs (as naturalistic as possible)
- 2) Fitted filters to behavioral data
→ More general, simpler, unfitted formalized Gabor filter bank

2 Developing a functional model of the grasshopper auditory pathway

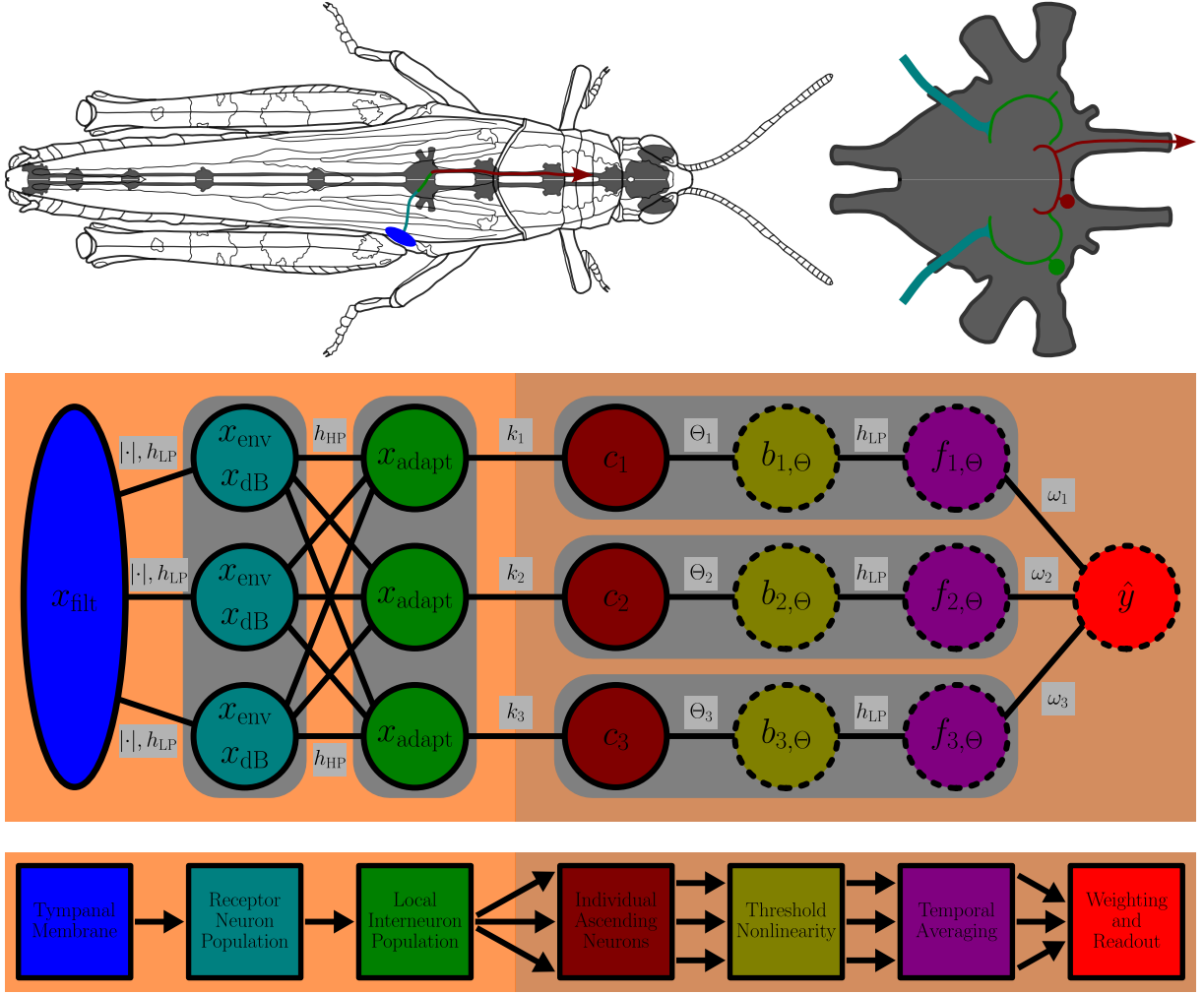


Figure 1: The auditory system of grasshoppers.

2.1 Population-driven signal pre-processing

Grasshoppers receive airborne sound waves by a tympanal organ at each side of the thorax. The tympanal membrane (Fig. 1) vibrates in response to incoming sound waves in a frequency-dependent manner: Vibrations of specific frequencies are focused on different membrane areas, while other frequencies are attenuated (Michelsen 1971; Windmill et al. 2008; Malkin et al. 2014). This mechanical resonance filter can be modelled by an initial band-pass filter

$$x_{\text{filt}}(t) = x(t) * h_{\text{BP}}(t), \quad f_{\text{cut}} = 5 \text{ kHz}, 30 \text{ kHz} \quad (1)$$

applied to the acoustic input signal $x(t)$.

”Pre-split portion” of the auditory pathway:

Tympanal membrane → Receptor neurons → Local interneurons

Similar response/filter properties within receptor/interneuron populations (Clemens, Kutzki, et al. 2011)

→ One population-wide response trace per stage (no ”single-cell resolution”)

Stage-specific processing steps and functional approximations:

Initial: Continuous acoustic input signal $x(t)$

Filtering of behaviorally relevant frequencies by tympanal membrane

→ Bandpass filter 5-30 kHz Extraction of signal envelope (AM encoding) by receptor population

→ Full-wave rectification, then lowpass filter 500 Hz

$$x_{\text{env}}(t) = |x_{\text{filt}}(t)| * h_{\text{LP}}(t), \quad f_{\text{cut}} = 500 \text{ Hz} \quad (2)$$

Logarithmically compressed intensity tuning curve of receptors

→ Decibel transformation

$$x_{\text{dB}}(t) = 10 \cdot \log_{10} \frac{x_{\text{env}}(t)}{x_{\text{ref}}}, \quad x_{\text{ref}} = \max[x_{\text{env}}(t)] \quad (3)$$

Spike-frequency adaptation in receptor and interneuron populations

→ Highpass filter 10 Hz

$$x_{\text{adapt}}(t) = x_{\text{dB}}(t) * h_{\text{HP}}(t), \quad f_{\text{cut}} = 10 \text{ Hz} \quad (4)$$

2.2 Feature extraction by individual neurons

"Post-split portion" of the auditory pathway:

Ascending neurons (AN) → Central brain neurons

Diverse response/filter properties within AN population (Clemens, Kutzki, et al. 2011)

- Pathway splitting into several parallel branches

- Expansion into a decorrelated higher-dimensional sound representation

→ Individual neuron-specific response traces from this stage onwards

Stage-specific processing steps and functional approximations:

Template matching by individual ANs

- Filter base (STA approximations): Set of Gabor kernels

- Gabor parameters: σ, ϕ, f → Determines kernel sign and lobe number

$$k(t) = e^{-\frac{t^2}{2\sigma^2}} \cdot \sin(2\pi ft + \phi) \quad (5)$$

→ Separate convolution with each member of the kernel set

$$c_i(t) = x_{\text{adapt}}(t) * k_i(t) = \int_{-\infty}^{+\infty} x_{\text{adapt}}(\tau) \cdot k_i(t - \tau) d\tau \quad (6)$$

Thresholding nonlinearity in ascending neurons (or further downstream)

- Binarization of AN response traces into "relevant" vs. "irrelevant"

→ Shifted Heaviside step-function $H(c_i - \Theta_i)$ (or steep sigmoid threshold?)

$$b_{i,\Theta}(t) = \begin{cases} 1, & c_i(t) > \Theta_i \\ 0, & c_i(t) \leq \Theta_i \end{cases} \quad (7)$$

Temporal averaging by neurons of the central brain

- Finalized set of slowly changing kernel-specific features (one per AN)
- Different species-specific song patterns are characterized by a distinct combination of feature values → Clusters in high-dimensional feature space
- Lowpass filter 1 Hz

$$f_{i,\Theta}(t) = b_{i,\Theta}(t) * h_{\text{LP}}(t), \quad f_{\text{cut}} = 1 \text{ Hz} \quad (8)$$

3 Two mechanisms driving the emergence of intensity-invariant song representation

3.1 Logarithmic scaling & spike-frequency adaptation

Envelope $x_{\text{env}}(t) \xrightarrow{\text{dB}}$ Logarithmic $x_{\text{dB}}(t) \xrightarrow{h_{\text{HP}}(t)}$ Adapted $x_{\text{adapt}}(t)$

- Rewrite signal envelope $x_{\text{env}}(t)$ (Eq. 2) as a synthetic mixture:

- 1) Song signal $s(t)$ ($\sigma_s^2 = 1$) with variable multiplicative scale $\alpha \geq 0$
- 2) Fixed-scale additive noise $\eta(t)$ ($\sigma_\eta^2 = 1$)

$$x_{\text{env}}(t) = \alpha \cdot s(t) + \eta(t), \quad x_{\text{env}}(t) > 0 \quad \forall t \in \mathbb{R} \quad (9)$$

- Signal-to-noise ratio (SNR): Ratio of variances of synthetic mixture $x_{\text{env}}(t)$ with ($\alpha > 0$) and without ($\alpha = 0$) song signal $s(t)$, assuming $s(t) \perp \eta(t)$

$$\text{SNR} = \frac{\sigma_{s+\eta}^2}{\sigma_\eta^2} = \frac{\alpha^2 \cdot \sigma_s^2 + \sigma_\eta^2}{\sigma_\eta^2} = \alpha^2 + 1 \quad (10)$$

Logarithmic component:

- Simplify decibel transformation (Eq. 3) and apply to synthetic $x_{\text{env}}(t)$
- Isolate scale α and reference x_{ref} using logarithm product/quotient laws

$$\begin{aligned} x_{\text{dB}}(t) &= \log \frac{\alpha \cdot s(t) + \eta(t)}{x_{\text{ref}}} \\ &= \log \frac{\alpha}{x_{\text{ref}}} + \log \left[s(t) + \frac{\eta(t)}{\alpha} \right] \end{aligned} \quad (11)$$

→ In log-space, a multiplicative scaling factor becomes additive

→ Allows for the separation of song signal $s(t)$ and its scale α

→ Introduces scaling of noise term $\eta(t)$ by the inverse of α

→ Normalization by x_{ref} applies equally to all terms (no individual effects)

Adaptation component:

- Highpass filter over $x_{\text{dB}}(t)$ (Eq. 4) can be approximated as subtraction of the local signal offset within a suitable time interval T_{HP} ($0 \ll T_{\text{HP}} < \frac{1}{f_{\text{cut}}}$)

$$x_{\text{adapt}}(t) \approx x_{\text{dB}}(t) - \log \frac{\alpha}{x_{\text{ref}}} = \log \left[s(t) + \frac{\eta(t)}{\alpha} \right] \quad (12)$$

Implication for intensity invariance:

- Logarithmic scaling is essential for equalizing different song intensities
 - Intensity information can be manipulated more easily when in form of a signal offset in log-space than a multiplicative scale in linear space
- Scale α can only be redistributed, not entirely eliminated from $x_{\text{adapt}}(t)$
 - Turn initial scaling of song $s(t)$ by α into scaling of noise $\eta(t)$ by $\frac{1}{\alpha}$
- Capability to compensate for intensity variations, i.e. selective amplification of output $x_{\text{adapt}}(t)$ relative to input $x_{\text{env}}(t)$, is limited by input SNR (Eq. 10):
 - $\alpha \gg 1$: Attenuation of $\eta(t)$ term → $s(t)$ dominates $x_{\text{adapt}}(t)$
 - $\alpha \approx 1$: Negligible effect on $\eta(t)$ term → $x_{\text{adapt}}(t) = \log[s(t) + \eta(t)]$
 - $\alpha \ll 1$: Amplification of $\eta(t)$ term → $\eta(t)$ dominates $x_{\text{adapt}}(t)$
 - Ability to equalize between different sufficiently large scales of $s(t)$
 - Inability to recover $s(t)$ when initially masked by noise floor $\eta(t)$
- Logarithmic scaling emphasizes small amplitudes (song onsets, noise floor)
 - Recurring trade-off: Equalizing signal intensity vs preserving initial SNR

3.2 Threshold nonlinearity & temporal averaging

Convolved $c_i(t) \xrightarrow{H(c_i - \Theta_i)}$ Binary $b_{i,\Theta}(t) \xrightarrow{h_{\text{LP}}(t)}$ Feature $f_{i,\Theta}(t)$

Thresholding component:

- Within an observed time interval T , $c_i(t)$ follows probability density $p(c_i, T)$
- Within T , $c_i(t)$ exceeds threshold value Θ_i for time T_1 ($T_1 + T_0 = T$)
- Threshold $H(c_i - \Theta_i)$ splits $p(c_i, T)$ around Θ_i in two complementary parts

$$\int_{\Theta_i}^{+\infty} p(c_i, T) dc_i = 1 - \int_{-\infty}^{\Theta_i} p(c_i, T) dc_i = \frac{T_1}{T} \quad (13)$$

→ Semi-definite integral over right-sided portion of split $p(c_i, T)$ gives ratio of time T_1 where $c_i(t) > \Theta_i$ to total time T due to normalization of $p(c_i, T)$

$$\int_{-\infty}^{+\infty} p(c_i, T) dc_i = 1 \quad (14)$$

Averaging component:

- Lowpass filter over binary response $b_{i,\Theta}(t)$ (Eq. 8) can be approximated as temporal averaging over a suitable time interval T_{LP} ($T_{\text{LP}} > \frac{1}{f_{\text{cut}}}$)
- Within T_{LP} , $b_{i,\Theta}(t)$ takes a value of 1 ($c_i(t) > \Theta_i$) for time T_1 ($T_1 + T_0 = T_{\text{LP}}$)

$$f_{i,\Theta}(t) \approx \frac{1}{T_{\text{LP}}} \int_t^{t+T_{\text{LP}}} b_{i,\Theta}(\tau) d\tau = \frac{T_1}{T_{\text{LP}}} \quad (15)$$

→ Temporal averaging over $b_{i,\Theta}(t) \in [0, 1]$ (Eq. 7) gives ratio of time T_1 where $c_i(t) > \Theta_i$ to total averaging interval T_{LP}

→ Feature $f_{i,\Theta}(t)$ approximately represents supra-threshold fraction of T_{LP}

Combined result:

- Feature $f_{i,\Theta}(t)$ can be linked to the distribution of $c_i(t)$ using Eqs. 13 & 15

$$f_{i,\Theta}(t) \approx \int_{\Theta_i}^{+\infty} p(c_i, T_{LP}) dc_i = P(c_i > \Theta_i, T_{LP}) \quad (16)$$

→ Because the integral over a probability density is a cumulative probability, the value of feature $f_{i,\Theta}(t)$ (temporal compression of $b_{i,\Theta}(t)$) at every time point t signifies the probability that convolution output $c_i(t)$ exceeds the threshold value Θ_i during the corresponding averaging interval T_{LP}

Implication for intensity invariance:

- Convolution output $c_i(t)$ quantifies temporal similarity between amplitudes of template waveform $k_i(t)$ and signal $x_{\text{adapt}}(t)$ centered at time point t

→ Based on amplitudes on a graded scale

- Feature $f_{i,\Theta}(t)$ quantifies the probability that amplitudes of $c_i(t)$ exceed threshold value Θ_i within interval T_{LP} around time point t

→ Based on binned amplitudes corresponding to one of two categorical states → Deliberate loss of precise amplitude information

→ Emphasis on temporal structure (ratio of T_1 over T_{LP})

- Thresholding of $c_i(t)$ and subsequent temporal averaging of $b_{i,\Theta}(t)$ to obtain $f_{i,\Theta}(t)$ constitutes a remapping of an amplitude-encoding quantity into a duty cycle-encoding quantity, mediated by threshold function $H(c_i - \Theta_i)$

- Different scales of $c_i(t)$ can result in similar T_1 segments depending on the magnitude of the derivative of $c_i(t)$ in temporal proximity to time points at which $c_i(t)$ crosses threshold value Θ_i

→ The steeper the slope of $c_i(t)$, the less T_1 changes with scale variations

→ If T_1 is invariant to scale variation in $c_i(t)$, then so is $f_{i,\Theta}(t)$

- Suggests a relatively simple rule for optimal choice of threshold value Θ_i :

→ Find amplitude c_i that maximizes absolute derivative of $c_i(t)$ over time

→ Optimal with respect to intensity invariance of $f_{i,\Theta}(t)$, not necessarily for other criteria such as song-noise separation or diversity between features

- Nonlinear operations can be used to detach representations from graded physical stimulus (to fasciliate categorical behavioral decision-making?):

1) Capture sufficiently precise amplitude information: $x_{\text{env}}(t)$, $x_{\text{adapt}}(t)$

→ Closely following the AM of the acoustic stimulus

2) Quantify relevant stimulus properties on a graded scale: $c_i(t)$

→ More decorrelated representation, compared to prior stages

3) Nonlinearity: Distinguish between "relevant vs irrelevant" values: $b_{i,\Theta}(t)$

→ Trading a graded scale for two or more categorical states

4) Represent stimulus properties under relevance constraint: $f_{i,\Theta}(t)$

→ Graded again but highly decorrelated from the acoustic stimulus

5) Categorical behavioral decision-making requires further nonlinearities

→ Parameters of a behavioral response may be graded (e.g. approach speed), initiation of one behavior over another is categorical (e.g. approach/stay)

- 4 Discriminating species-specific song patterns in feature space
- 5 Conclusions & outlook